Homework 7: Due SUNDAY May 20th, 2018 at ***3 pm***

First, do these problems. These will be graded for completion. You must show enough work to convince the grader that you didn’t just copy the answer.

- 7.5: 1, 4, 6, 9, 14
- 7.6: 1, 3b, 15, 19, 22c (for just the first vector field.)

You might have fun trying to figure out how to set up 7.5:15, but I won’t make you do it. (Hint: Use spherical coordinates.)

Then do these problems. These will be graded for correctness.

1. In class, we discussed how the idea of flux also extends to 2 dimensions by essentially the same formula, \[ \text{flux} = \int_C \vec{F} \cdot \hat{n} \, ds. \] For a function \( f(x) \), we can define the path defined by that function by \( (t, f(t)) \), kind of like how we did it for parameterized surfaces. Find a formula for the flux across a graph in 2 dimensions, in the spirit of 7.6, formula (4). Use the upward pointing normal.

2. As we discussed in class, if \( \vec{F} \) is a velocity field, then the integral \( \int \int_S \vec{F} \cdot \hat{n} \, dA \) represents the total flow of the field \( \vec{F} \) through the surface \( S \). Explain why that integral and that integrand are the correct choices to calculate that quantity.

3. What is an orientation of a surface? What difference does it make if you pick one versus the other?

4. Let \( S \) be \([\text{the surface of}]\) a sphere, and \( F \) represent the a steady velocity flow of water. It turns out, as we’ll see in chapter 8, that \( \int \int_S F \cdot dS \) must be zero, no matter what \( F \) is, as long as it is a steady flow. However, we can explain why this is true even without chapter 8.

   What does the integral physically represent? Explain what it would mean if that integral were positive, and why that is impossible. Make a similar argument about the integral being negative. Thus, it has to be zero. (This should just be an intuitive argument, not a rigorous one.)