6.1.1. a. both  
    b. neither  
    C. both  
    d. neither.

6.1.2. If \( T \) is one-to-one, then, since \( T(b) = 0 \), \( x = 0 \) is the only value of \( x \) for which \( T(x) = 0 \), i.e., for which \( Ax = 0 \). That implies \( A \) is invertible, so \( \det A \neq 0 \).

If \( \det A \neq 0 \), then ... Suppose \( T(x) = T(x') \).

Then \( Ax = A x' \).

Since \( A \) is invertible, \( A^{-1}Ax = A^{-1}A x' \)

\( x = x' \).

Thus, we showed that if \( T(x) = T(x') \), then \( x = x' \). Thus \( T \) is one-to-one.

6.1.3. \( T(x) \) being onto means that \( T(x) = y \) has a solution for any \( y \).

That means that \( Ax = y \) has a sol'n.

But \( Ax = y \) always has a sol'n if and only if \( A \) is invertible.

Thus \( \det A \neq 0 \).

1.4.1. a. flip over \( xy \) plane.
    b. flip over \( xy \) plane, then rotate by \( 180^\circ \) in \( \theta \) direction.
    C. \( z \) stays unchanged.

\( (r, \theta) \rightarrow (-r, \theta - \frac{\pi}{4}) \)

5. a. rotate in \( \theta \) direction by \( \pi \).
    b. flips over \( xy \) plane!
    C. rotates in \( \theta \) direction by \( \frac{\pi}{2} \), then doubles the length.

   rotate clockwise by \( \frac{\pi}{4} \), then find antipodal (opposite) point.
   This is equivalent to just rotating by \( 3 \frac{\pi}{4} \), counter-clockwise.
6.2.3: \[ \iint e^{x^2+y^2} \, dx \, dy = \int_0^1 \int_0^2 \pi r e^{-r^2} \, dr \, d\theta = \quad \]

6.2.2b: This region is almost like a sphere, but \( x, y, z \) are all forced to be positive. Thus the region is the portion of the sphere in the 1st Octant. So

\[
\int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \frac{r}{1+r^2} r^2 \sin \phi \, dr \, d\phi \, d\theta = \quad \]

6.2.31: I could do this in 3 integrals, but I'll change coords instead. The labels to the left are what I want the points to be in the new coords. Note that I cannot have \( T(0,0) = (0,0) \), but I want \( T(0,0) = (1,0) \).

So, I'll take \( T(u,v) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \).

Using \( T(0,0) = (1,0) \)
\( T(3,0) = (4,3) \)
\( T(0,1) = (0,1) \) in order, I get \( T(u,v) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \),

i.e., \( T(u,v) = (u-v+1, u+v) \)

Thus \( \| DT \| = 2 \)

\[ \iint_B (x+y) \, dx \, dy = \int_0^1 \int_0^3 (u-v+1+u+v) \, du \, dv = 2 \int_0^1 \int_0^3 (2u+1) \, du \, dv \quad \]
1. Way 1: vector field. Essentially, at each point of the domain, we get out a vector in $\mathbb{R}^2$, so $T$ could represent a vector field for the domain $D^*$. 

Way 2: transformation. We can think of $T$ as stretching $D^*$ to transform it into the region $D$.

Way 3: As coordinates. $D^*$ is simply coordinates for $D$, giving each point in $D$ a new name, where $T(u,v) = (x,y)$ tells you that the new name for $(x,y)$ is $(u,v)$.

2. Both are simply interpretations of $\iint_{\text{region}} f \, dA$, just in terms of either rectangular coords, for the left one, or the new coords for the right one. The only tricky part is $dA = dx\,dy = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$. To calculate a bit of area, we take a bit of coordinate area, $du\,dv$, then see how it transforms.

$T$ transforms $(du,0)$ into $(\frac{dx}{du}, 0)$ and $(dv,0)$ into $(\frac{dy}{dv}, 0)$.

The area of the parallelogram is thus the det. of those vectors which is $\left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$. 

$\iint_{\text{region}} f \, dA$
3. If $T$ is not injective, then two points in $D^*$ get mapped onto a single point in $O$. Thus, that point has two new names. Which should we use? If we’re not careful, we’ll double count area, which is bad.

If $T$ is not surjective, then $T$ doesn’t cover all of $O$. That means not every point in $O$ has a new name, so $T$ is not doing its job of new coords very well. It only gives new coords to part of $O$. 