Homework 3: Due April 23rd, 2018

First, do these problems. These will be graded for completion. You must show enough work to convince the grader that you didn’t just copy the answer.

- 6.1: 1, 4, 12, 13,
- 1.4: 1, 4, 5, 8b, 10
- 6.2: 1, 3, 15, 23, 26, 31
- Calculate $\int_{-\infty}^{\infty} e^{-x^2} \, dx$ exactly. (I give you permission to look this question up on the internet. If you want to try to figure it out on your own, look at 6.2:3 and the hint (just the hint) in 5.4:18.)

Then do these problems. These will be graded for correctness.

1. Explain two different ways to geometrically interpret a map $T : D^* \to D$, where $D, D^* \subset \mathbb{R}^2$, assuming $T$ is bijective and $C^1$.

2. The change of variables theorem, as given in the book, says that if $T : D^* \to D$ is $C^1$ and one-to-one, and $T(D^*) = D$, then if $f$ is integrable,

$$
\int\int_D f(x,y) \, dxdy = \int\int_{D^*} f(u(x,y), v(x,y)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dudv.
$$

Similar to how we discussed it in class, explain why this theorem is true. Your explanation should include an explanation of why $\left| \frac{\partial(x,y)}{\partial(u,v)} \right|$ is the correct factor.

3. Explain why $T : D^* \subset \mathbb{R}^2 \to D \subset \mathbb{R}^2$ should generally be injective and surjective if we want $T$ to represent a choice of coordinates for the region $D$. 