5.3.2. a.

\[ \text{Diagram of a quadrant with shaded area} \]

4. b.

\[ \text{Diagram of intersecting axes} \]

12.

\[ \int_D (\cos(y)) \, dx \, dy = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} (\cos(y)) \, dy \, dx \]

19. Integral is \[ \int_a^b \int_{-\phi(x)}^{\phi(x)} f(x, y) \, dy \, dx. \] The inner integral is the integral of an odd function over a symmetric domain. Thus, the inner integral is always 0. Thus, it is overall 0.

5.4.1.b

\[ \text{Diagram of a region with shaded area} \]

\[ \int_0^{\pi} \int_0^{\pi/2} (\pi - \arcsin(y)) \, dx \, dy \]

Notice that \( \arcsin(y) \) is not the correct upper limit since we need angles between \( \frac{\pi}{2} \) and \( \pi \).
5.4.5: \[ \int_{y}^{x} x^2 \, dy = \int_{0}^{1} e^{-x^3} \, dy \, dx = \int_{0}^{1} e^{-x^3} \, dx \]

5.4.6 a. True.

b. True.

C. True.

d. True.

7. Area is \( \frac{4}{\pi} \), and \(-1 \leq \sin(x+y) \leq 1\), so \(\varepsilon \leq f \leq e\).

So, by the inequality in the book, we get it.

19. Let \( g(u,v) = \int_{a}^{u} \int_{c}^{d} f(v,y,z) \, dz \, dy \)

Then we want to calculate \( \frac{d}{dx} (g(x,x)) = \frac{dg}{du} \frac{du}{dx} + \frac{dg}{dv} \frac{dv}{dx} \).

\( \frac{du}{dx} = \frac{dv}{dx} = 1 \), since \( u = v = x \).

\( \frac{dg}{du} = \int_{c}^{d} f(v,y,z) \, dz \) by the fund. Thm of calc.

\( \frac{dg}{dv} = \int_{a}^{u} \int_{c}^{d} \frac{df}{dv} (v,y,z) \, dz \, dy \).

Putting these together, we get what the book has.
5.5.1. a. \( \rightarrow i \)
   b. \( \rightarrow i \)
   c. \( \rightarrow iii \)
   d. \( \rightarrow iv \).

\[ z = \int_0^1 \int_0^{\pi} \int_0^x \sin x \, dz \, dy \, dx = \ldots \]

24. a. By 1, it's the same as 1c, so see that_pic (iii)
   b. \[ \int \int \int f \, dx \, dy \, dz \]
      I'm pretty sure...

26. \[ \int \int \int f \, dy \, dx \, dz \]

27. Intersection of cylinder, sphere, upper space,

\[ \int \int \int f \, dz \, dy \, dx \]

\[ -1 \leq z \leq \sqrt{1-x^2} \]

\[ -\sqrt{1-x^2} \leq z \leq \sqrt{1-x^2} \]
1. The units of $\int \int \int_{R} c(x,y,t) \, dx \, dy \, dt$ are $\frac{cm}{day} \cdot Km \cdot Km \cdot days = Km^2 \cdot cm$ and it represents the total rainfall over CA in 2017.

The "volume" has the units of $dx \, dy \, dt = Km \cdot Km \cdot days$, while the units for the average are $\frac{Km^2 \cdot cm}{Km^2 \cdot day} = \frac{cm}{day}$.

2. If $f$ is 0 on part of the domain, and 1 on the rest, the average value will be between 0 and 1, but it will never have a value other than 0 or 1.

3. $\frac{1}{\text{Vol}(B_n)} \int_{B_n} f \, dv$ gives the average value of $f$ over the small ball $B_n$. When we take the limit, we are finding the average value over smaller and smaller balls. If $f$ is continuous, the values very near $(x_0, y_0, z_0)$ must get arbitrarily close to $f(x_0, y_0, z_0)$. Thus, the average must get closer and closer to that value. Thus, in the limit, the average over small balls must approach the value at the center point, as claimed.

4. We want to show $2 \int_{a}^{b} \int_{x}^{b} f(x) f(y) \, dy \, dx = \int_{a}^{b} f(x) f(y) \, dy \, dx = \left( \int_{a}^{b} f(x) \, dx \right)^2$

The first integral is over this domain:

While the second is over the square version. So the first equality makes sense if

The integral over the top triangle is the same as the integral over the lower triangle. This is true b/c the integrand is symmetric in $x$ and $y$, i.e., it stays the same if you switch $x$ and $y$.

(continued)
The last equality is a calculation

\[
\int_a^b \int_a^b f(x) f(y) \, dy \, dx = \int_a^b f(x) \left( \int_a^b f(y) \, dy \right) \, dx \quad \text{(since } f(x) \text{ doesn't depend on } y) \]

\[
= \int_a^b f(y) \, dy \int_a^b f(x) \, dx \quad \text{(since } \int_a^b f(y) \, dy \text{ is a constant.)}
\]

\[
= \left( \int_a^b f(x) \, dx \right)^2 \quad \text{(since } \int_a^b f(y) \, dy = \int_a^b f(x) \, dx \text{)}
\]