Homework 2: Due April 16th, 2018

First, do these problems. These will be graded for completion. You must show enough work to convince the grader that you didn’t just copy the answer.

- 5.3: 2, 4b, 12, 19
- 5.4: 1d, 5, 6, 7, 19
- 5.5: 1, 2, 19, 24, 26, 27

Then do these problems. These will be graded for correctness.

1. Let $r(x,y,t)$ be the rate of rainfall (in cm/day) at the location $(x,y)$ (measured in km), $t$ days after Jan 1st, 2017. Let $R$ be the domain defined as California $\times [0,365]$. What are the units of $\iiint_R r(x,y,t) \, dx \, dy \, dt$, and what does that quantity represent?

To find the average value of $r(x,y,t)$ over that domain, we would need to divide this integral by a constant, the “volume” of $R$. What are the units of this “volume”? What are the units of the average value?

2. Sketch and/or describe a region $D$ and a function $f(x,y)$, such that $f(x,y)$ never achieves its average value over $D$. (In other words, such that the mean value theorem doesn’t work.)

3. Let $f$ be continuous and let $B_\epsilon$ be the ball of radius $\epsilon$ centered at the point $(x_0,y_0,z_0)$. Let $\text{vol}(B_\epsilon)$ be the volume of $B_\epsilon$. Explain why, intuitively,

$$\lim_{\epsilon \to 0} \frac{1}{\text{vol}(B_\epsilon)} \iiint_{B_\epsilon} f(x,y,z) \, dV = f(x_0,y_0,z_0).$$

If you are a math major, you should also try to prove this rigorously, though you don’t have to turn that part in.

4. 5.4.18. In addition, explain why or why not.