Discussion 9 Solutions

1. Total flow along a path \( \int_C \mathbf{F} \cdot d\mathbf{s} = \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} \), for any surface \( S \), with the boundary of \( S \), \( dS \), being the path. Since the given path is a closed curve, we can do this:

\[
\text{curl} \mathbf{F} = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
1 & 1 & 0 \\
2 & 0 & 1
\end{vmatrix} = (0, 0, 0), \text{ so } \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = 0!
\]

So the flow along the path is zero.

2. \( \sqrt{3} \) by Pythagorean Thm.

I've drawn the orientation of the boundary induced by the outward normal on \( S \). The path for the boundary could thus be parametrized by \( \gamma(t) = (\sqrt{3} \cos t, \sqrt{3} \sin t, 1) \) on [0, 2\pi],

\[
\gamma'(t) = (-\sqrt{3} \sin t, \sqrt{3} \cos t, 0)
\]

\[
\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_0^{2\pi} \left( \frac{1}{4}, 0, \sin(x^2y^2) \right) \cdot \gamma'(t) \, dt
\]

\[
= \int_0^{2\pi} \frac{1}{\sqrt{3} \sin t} \cdot (\sqrt{3} \sin t) + 0 + 0 \, dt = 2\pi
\]

(Note: \( F \) is \( C^1 \) on the surface, though not on all \( \mathbf{C} \), so Stokes' Thm still holds.) So answer is \( 2\pi \). Since my calculation used the wrong orientation.

1. \( \int_C \mathbf{F} \cdot d\mathbf{s} \) is the flow of \( \mathbf{F} \) along the curve \( C \). If \( \mathbf{F} \) is constant, this should be 0 for a closed curve because the flow with \( \mathbf{F} \) on one side should cancel out the flow against \( \mathbf{F} \) on the other side, since the curve is closed, and the field is constant.

Also, using Stokes' Thm, \( \text{curl} \mathbf{F} = 0 \), so \( \int_C \mathbf{F} \cdot d\mathbf{s} = \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = 0. \)