Discussion 2 Solutions

1. The base has this shape:

It then ranges from \( z=0 \) to \( z=1 \).

\[
\text{mass} = \int_{-2}^{1} \int_{0}^{x} \int_{-rac{2}{x^2}}^{z(x^2)} dz \, dx \, dz = \int_{0}^{1} \int_{-2}^{x} \left[ \frac{2z(x^2)}{2z} \right]_{-rac{2}{x^2}}^{z(x^2)} dx \, dz
\]

\[
= 2 \int_{0}^{1} \int_{-2}^{x} \left[ 8 + 4x - 2x^2 - \frac{3}{2} x^3 \right] dx \, dz
\]

\[
= 2 \int_{0}^{1} \left[ 64 - 3x^2 \right]_{-2}^{x} dz
\]

\[
= 2 \int_{0}^{1} \frac{64}{3} dz = \frac{64}{3} \text{ Kg}
\]

Volume:

\[
\int_{0}^{1} \int_{-2}^{x} \int_{-rac{2}{x^2}}^{z(x^2)} dy \, dx \, dz = \int_{0}^{1} \int_{-2}^{x} 8 - 2x^2 \, dx \, dz = \int_{0}^{1} \frac{64}{3} \, dz = \frac{64}{3} \text{ cm}^3
\]

So average density is \( \frac{64/3}{64/3} = 1 \text{ Kg/cm}^3 \)

2. Two kinds of regions:

\( \ln(y+1) \leq x \leq 1 \) on top

\( \ln(y+1) \leq x \leq \arcsin(y) \) on bottom.

Changes at \( y = \sin(1) \)

\[
e^x - 1 = y \\
x = \ln(y+1) \\
\sin(x) = y \\
x = \arcsin(y)
\]

So:

\[
\int_{0}^{1} \int_{\ln(y+1)}^{\arcsin(y)} f(x) \, dx \, dy + \int_{\sin(1)}^{1} \int_{\ln(y+1)}^{e-1} f(x) \, dx \, dy
\]
Graded 1. \( dA \) represents a tiny bit of area, and the \( \int \int \int \) adds it up, so \( \int \int \int dA \) gives the total area.

As a repeated integral, we have something like \( \int_a^b \int_c^d \int_e^f \, dx \, dy \, dz \)

The inner integral adds up bits of \( x \) (\( dx \)) to get the total length of a slice. This length multiplied by a bit of \( y \) (\( dy \)) gives the area of a slice. Then we add up all those slice areas to get the total area, as in Cavalieri's principle.

The triple integral works the same way. \( \int_0^D \int_0^V \int_0^T dV \) adds up bits of volume \( dV \) to get total volume.

Also \( \int_a^b \int_c^d \int_e^f \, dx \, dy \, dz \)

works similarly as a slice of slices. The inner integral adds up bits of length, the second adds up bits of area, to get area of slices, and the outer adds up the volume of those slices to get the total volume.