Discussion 1: Question to be turned in.

Names: 1. Solution 2. 3. 4.

TA initial: ____________________

1. A system of linear equations with fewer equations than unknowns is sometimes called an **underdetermined system**. Suppose that such a system happens to be consistent. Explain why there must be an infinite number of solutions. As part of your explanation, give an example of an inconsistent underdetermined system of two equations and three unknowns. You should explain this \textit{both} in terms of the geometric intuition as well as in terms of the augmented matrix.

For a geometric interpretation, it is easiest to think about a concrete example, say when we have two equations with 3 unknowns. In this case, each eqn represents a plane. While the planes could be parallel, we know the system is consistent, so they must intersect. Two planes in 3 dim. always intersect along a line, and so there must be infinitely many solns.

Looking at the augmented matrix for such a system, we could have
\[
\begin{bmatrix}
1 & 2 & 3 & 1 \\
1 & 2 & 4 & 2
\end{bmatrix}.
\]
The RREF is \[
\begin{bmatrix}
1 & 2 & 3 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix},
\] so it is consistent. However, since the coefficient part of the matrix (the first 3 columns) has more columns than rows, there will always be at least one free variable, and so, if the system is consistent, it will have infinitely many solutions.

If the 4 was a 3, \[
\begin{bmatrix}
1 & 2 & 3 & 1 \\
1 & 2 & 3 & 2
\end{bmatrix},
\] the system would be inconsistent since the RREF would be \[
\begin{bmatrix}
1 & 2 & 3 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]