Many results in mathematical relativity, both for the initial data problem and for the evolution problem, rely on the existence of a constant mean curvature (CMC) Cauchy surface. However, it is known that not all spacetimes have any CMC Cauchy surfaces. This is a serious problem, especially because it is not even known whether spacetimes with CMC Cauchy surfaces are in any sense generic. In this primarily expository talk, we will discuss known results about the existence (and non-existence) of CMC Cauchy surfaces, and make several conjectures concerning their existence and generality. (Received September 25, 2017)