The relationship between the topology of spacetime and the occurrence of singularities (causal geodesic incompleteness) is a topic of long-standing interest. In this talk we focus on the cosmological setting: We consider globally hyperbolic spacetimes with compact Cauchy surfaces under assumptions compatible with the presence of a positive cosmological constant. More specifically, for 3+1 dimensional spacetimes which satisfy the null energy condition and contain a future expanding compact Cauchy surface, we establish a precise connection between the topology of the Cauchy surfaces and the occurrence of past singularities. In addition to (a refinement of) the Penrose singularity theorem, the proof makes use of certain fundamental existence results for minimal surfaces and of some recent advances in the topology of 3-manifolds. (Received September 26, 2017)