Name: ______________________________ **SID:** ______________

Instructor: ____________________ **Sec. No:** _____ **Sec. Time:** _____

Math 454 Partial Differential Equations.
Sample Midterm 1 Examination
Oct. 11, 2010

Turn off and put away your cell phone.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1		
2		
3		
4		
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8		
9		
Σ		

1. Solve for $u(x, y)$:

   (a) $u_{xx} = 0$.

   (b) $u_{xx} + u = 0$.

   (c) $u_{xy} = 0$.

2. Verify that $u_n(x, y) = \sin(nx)\sinh(ny)$ solves $u_{xx} + u_{yy} = 0$ for all $n > 0$.

3. Show that $u(x, y) = f(bx - ay)$ solves $au_x + bu_y = 0$, for a, b not both zero and where f is any differentiable function of one variable.

4. Sketch the discrete frequency spectrum of the function whose Fourier series expansion is given by

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{\sin(n\pi x)}{n}.$$

5. (a) Find the Fourier sine series expansion of $f(x) = 10$ on the interval $0 \leq x \leq L$.

   (b) Is the expansion valid at $x = 0$, $x = L$? If not, does this contradict Fourier's theorem?

6. For each statement, circle **T** if it is ***always*** True; circle **F** if it is ***ever*** False. If the statement is false then give a counter example or explain why the statement is incorrect. If the statement is True no justification is needed.

(T F) All periodic functions have a Fourier series expansion.

(T F) The function $f(x) = x^{1/5}$ is piecewise smooth on the interval $(-\pi, \pi)$.

(T F) The function $g(x) = \dfrac{\sin x}{x}$ is piecewise smooth on the interval $-2 \le x \le 2$.

7. Using the orthogonality property of sines and cosines, show that

$$\int_{-L}^{L} e^{\frac{im\pi x}{L}} e^{\frac{-in\pi x}{L}}\, dx = \gamma,$$

where $\gamma = 0$ when $n \neq m$ and $\gamma = 2L$ when $n = m$.

8. Consider the diffusion-convection equation $u_t = \alpha^2 u_{xx} - \gamma u_x$. Assume a solution of the form

$$u(x,t) = e^{\frac{\gamma(x-.5\gamma t)}{2\alpha^2}} w(x,t).$$

Find the PDE for $w(x,t)$.

9. Solve the one dimensional wave equation using separation of variables.

$$
\begin{aligned}
u_{tt} &= c^2 u_{xx}, \quad 0 < x < L, \\
u(x,0) &= f(x), \quad u_t(x,0) = g(x), \quad 0 \le x \le L, \\
u(0,t) &= 0 = u(L,t), \quad t > 0,
\end{aligned}
$$

10. Sample problems 1-9 should take you 1.5 to 2 hours to complete. I will not give you this long of an exam. For more sample problems, redo the problems assigned in the homework page. Review your lecture notes. You are responsible for the materials in the textbook Lessons 1 to 14. You are also responsible for materials covered in class whether or not it is in the textbook. In particular, please review all the theorems and concrete examples I gave in class. Problems involving Laplace transforms will be given. Also, study problems involving nonhomogenous boundary conditions.