Final Review Hints

Thomas Laetsch

1

1. $\Delta x = (2-1)/4$ and we have LHS $= (f(1) + f(5/4) + f(6/4) + f(7/4))\Delta x$ where $f(x) = x^2$.
   RHS $= (f(5/4) + f(6/4) + f(7/4) + f(2))\Delta x$
   TRAP = (RHS + LHS)/2

2. The difference between the RHS and LHS is $\Delta x(f(2) - f(1))$, which we want to set equal to .3 to solve for Δx.

2

1. If $p \neq -1$ then we just have $x^{p+1}/(p+1) + C$. If $p = -1$, then we have $\ln|x| + C$.

2. By parts with $u = x, dv = e^x$.

3. By parts twice, the first time with $u = x^2, dv = e^x$ and then it will reduce down to an integral that looks like the previous problem.

4. By parts with $u = ln(x), dv = x$

5. u-Substitution with $u = ln(x)$

6. u-Substitution with $u = ln(x)$ - same substitution as the last problem

7. u-Substitution with $u = 1 - x^2$

8. Trig-Sub with $x = sin(\theta)$ (or you could of course set $x = cos(\theta)$ and it'll work too).

9. u-Substitution with $u = tan(x)$

10. This one you first should turn $tan(x) = sin(x)/cos(x)$ and simplify to get something in the integrand like $sin(x)/cos^2(x)$. Then u-Substitution with $u = cos(x)$.

11. Partial fractions, since $1 - x^2 = (1-x)(1+x)$.

12. u-Substitution with $u = 1 + x^2$

13. This is a tricky one, but not terrible. Trig-Sub with $\sqrt{2}x = sin(\theta)$. Before you do the trig sub, factor out the 4 in the square root in the bottom to see why we want to consider this trig sub. After you make the trig sub, you'll have something in the integrand that looks like $1/sin(\theta)d\theta$ - it should be a constant times this value. Remember then to have $1/sin(\theta) = sin(\theta)/sin^2(\theta) = sin(\theta)/(1 - cos^2(\theta))$. Now, make a u-substitution with $u = cos(\theta)$.

14. Partial fractions since $x^2 + 5x + 6 = (x + 2)(x + 3)$

3

1. Look at number 8 in the previous section

2. To first solve the integral, realize that it is improper (at both 0 and ∞). Now, solve it by parts with $u = ln(x), dv = 1$. It will diverge as it goes to ∞.

3. To solve the integral look at problem 5 in the last section. It will diverge as it goes to ∞.

4. This is just the second fundamental theorem. Ans: $ln(x)/sin(x)$.

5. First, split up the integral to look like $\frac{d}{dx}\int_1^{cos(x)} ln(t)dt - \frac{d}{dx}\int_1^{sin(x)} ln(t)dt$. Now solve both using the chain rule where in the first integral you will set $f(x) = \int_1^x ln(t)dt$ and $g(x) = cos(x)$ so that we have $\frac{d}{dx}\int_1^{cos(x)} ln(t)dt = \frac{d}{dx}f(g(x)) = f'(g(x))g'(x) = -ln(cos(x))sin(x)$. Same idea for the second integral with the difference that $g(x) = sin(x)$.

6. Here there are a couple places where this integral is improper (at 0 and 1). So you need to split up the integral: $\int_0^1 1/(x\sqrt{1 - x^2}) + \int_1^2 1/(x\sqrt{1 - x^2})$. We solved a similar integral in the last section (number 13). This time it is going to be a trig sub again with $x = sin(\theta)$.

7. We have to consider the cases where $p \neq -1$ and $p = -1$ to get the antiderivative. (See problem 1 from the last section). If $p \geq -1$ this will diverge. If $p < -1$ it will converge.

4

1. Remember that $dv/dt = -g$ so that $v(t) = v_0 - gt$ (after solving for the constant by knowing that $v(0) = v_0$) and $dh/dt = v(t)$, so taking the antiderivative of both sides gives us $h(t) = h_0 + v_0 t - \frac{1}{2}gt^2$.

2. Setting up the problem as we did in the last one we would have to subtract v_0, that is $h(t) = h_0 - v_0 t - \frac{1}{2}gt^2$, $v(t) = -v_0 t - gt$.

5

1. Separate, so it looks like $\frac{dy}{y} = \frac{dx}{x}$ and then integrate both sides.
2. Add P^2 to both sides and then separate so it looks like $\frac{dP}{1+P^2} = dt$ and integrate both sides.
3. Separate, so it looks like $\frac{du}{u^2} = ydy$, integrate, and then use the initial condition to solve for the constant of integration.
4. The rate of change of the pressure P is directly proportional to the time and inversely prop. to the pressure. So, $\frac{dP}{dt} = k\frac{t}{P}$ where k is the constant of proportionality. Split this up to get $PdP = tdt$ and integrate. Use the initial value to solve for the constant.

6

1. $\int_0^1 \pi(x^2)^2 dx$
2. $\int_0^1 \pi((x^2+1)^2 - 1^2)dx$
3. $\int_0^1 \pi((1^2 - (\sqrt{y})^2)dy$
4. I'm not sure which region you interpret this question as asking for. I meant the region in 1. So, $\int_0^1 x^2 dx$.
5. $\int_0^1 \pi[(x^2+1)^2 - (x^3+1)^2]dx$
6. $\int_0^1 x^2 - x^3 dx$