Final Review Questions

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Beginning stuff:

1. Approximate $\int_1^2 x^2 dx$ by a left hand sum and a right hand sum with four subdivisions. Also, give the value you would get using the trapezoid rule.
2. How many subdivisions would I need to ensure that my LHS is within .3 of the correct answer.

Find the general anti-derivatives:

1. $\int x^p dx$
2. $\int xe^x dx$
3. $\int x^2 e^x dx$
4. $\int x \ln(x) dx$
5. $\int \frac{\ln(x)}{x} dx$
6. $\int \frac{1}{x \ln(x)} dx$
7. $\int x\sqrt{1-x^2} dx$
8. $\int \sqrt{1-x^2} dx$ [Remember: $2cos^2(x) = 1 + cos(2x)$, $2sin^2(x) = 1 - cos(2x)$]
9. $\int \tan^2(x) \sec^2(x) dx$
10. $\int \frac{\tan^2(x)}{\sin(x)} dx$
11. $\int \frac{x}{1-x^2} dx$
12. $\int \frac{x}{1+x^2} dx$
13. $\int \frac{1}{x\sqrt{4-8x^2}} dx$

14. $\int \frac{1}{x^2+5x+6}dx$

Find the following integrals and derivatives. If they diverge, say so:

1. $\int_0^1 \sqrt{1-x^2}dx$
2. $\int_0^\infty \ln(x)dx$
3. $\int_1^\infty \frac{\ln(x)}{x}dx$
4. $\frac{d}{dx}\int_1^x \frac{\ln(t)}{\sin(t)}dt$
5. $\frac{d}{dx}\int_{\sin(x)}^{\cos(x)} ln(t)dt$
6. $\int_0^2 \frac{1}{x\sqrt{1-x^2}}dx$
7. $\int_1^\infty x^p dx$ where p is a constant.

Suppose that Jane was standing on top of a building of height h_0 meters and throws a ball upward at an initial speed of v_0.

1. Give the equations for height and velocity of the ball as a function of time (in seconds).
2. What would change in the equations if the initial velocity was downward?

Solve the following differential equations:

1. $\frac{dy}{dx} = \frac{y}{x}$
2. $\frac{dP}{dt} - P^2 = 1$
3. $\frac{du}{dy} = u^2 y$, $u(0) = 1$
4. Suppose that the rate of change of the pressure in a room is directly proportional to the time and inversely proportional to the pressure. Knowing that you can never have a negative pressure, find the pressure P in terms of time t if $P(0) = 100$atm (atm is just a unit of pressure - called atmospheres) and the constant of proportionality is 2.

Find the following areas or volumes of rotation:

1. The volume created by rotating the region bounded by $y = x^2, y = 0, x = 0$, and $x = 1$ around the x-axis.
2. The same region in the previous problem, but now revolved around the line $y = -1$.
3. The same region as the previous, but now revolved around the y-axis.
4. The area of the previous bounded region.
5. The volume of the region bounded by $y = x^2, y = x^3, x = 0$, and $x = 1$ revolved around the line $y = -1$.
6. The area of the bounded region in the last problem.