Thermally-Driven Rare Events and Action Minimization

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Thermal or stochastic effects are prevalent in physical, chemical, and biological systems. Particularly in small systems, noise can overpower the deterministic dynamics and lead to "rare events," events which would never be seen in the absence of noise. One example is the thermally-driven switching of the magnetization in small memory elements. Wentzell-Freidlin large deviation theory is a mathematical tool for studying rare events. It estimates their probability and also the "most likely switching pathway," which is the pathway in phase space by which rare events are most likely to occur. We explain how large deviation theory and concepts from stochastic resonance may be applied to analyze thermally-activated magnetization reversal in the context of the spatially uniform Landau-Lifschitz-Gilbert equations. The time-scales of the experiment are critical. One surprising and physically relevant result is that in multiple-pulse experiments, nonconvential "short-time switching pathways" can dominate. The effect is dramatic: the usual pathway (connected with the Arrhenius-law) underestimates the probability of switching by an exponential factor.

An advantage of the method via large deviation theory is that it generalizes to systems with spatial variation. To discuss the complications and richness that emerge when spatial variation is taken into account, we consider the (simpler) Allen-Cahn equation. In this context, the rare event of interest is phase transformation from $u = -1$ to $u = +1$, and the most likely switching pathway is a pathway through function space. A natural reduced problem emerges in the "sharp-interface limit." We give a brief overview of some results (rigorous in $d = 1$, heuristic in $d > 1$).

The first part of the talk is joint work with Bob Kohn and Eric Vanden–Eijnden. The second part includes work that is also joint with Felix Otto and Yoshihiro Tonegawa.